

Name of college & Inst. collage,

J-Banu

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DATE: 14-08-2022

Dept: Double major

Topic: Asymptotes

Class: B.Sc.T (Hons) (Differential Calculus)

Time: 10.15 A.M to ~~11.00 A.M~~ 11.00 A.M

11.00 A.M to 11.

11.45 A.M to

Date: 14-08-2022

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Oblique Asymptotes of Algebraic Curves

Solution →

Let $F(x, y)$ be used to denote rational algebraical expression which contains terms of all and lower but of no higher degree.

① If the Equation of a curve of the n th degree be written in the form

$(ax+by+c) F_{n-1} + F_{n-2} = 0$ then

the equation

$$ax+by+c + \lim_{y \rightarrow -\infty, x \rightarrow \infty} \frac{F_{n-1}}{F_{n-2}} = 0$$

represents an asymptote of the curve.

② If the equation of the curve of n th degree can be put into the form

$(ax+by+c)^2 F_{n-2} + F_{n-3} = 0$, then the equations

$$ax+by+c = \pm \lim_{x \rightarrow \infty} \frac{F_{n-3}}{F_{n-2}}$$

represents a pair of parallel asymptotes of the curve, where x and y tends to infinity in the ratio $\frac{x}{y} = -b/a$ on the right hand side.

Problem based on when the Equation of the form

$$(a_1x + b_1y + c_1)P_{n-1} + g_{n-2} = 0$$

Ex:- Find the asymptotes of the curve.

$$(x+y)(x^4+y^4) = a(x^4+a^4)$$

Solution - The Equation of the curve is of fifth degree and this can be written as follows

$$(x+y)P_4 + Q_4 = 0$$

\therefore Asymptote will be parallel to $x+y=0$.

Thus the Equation of Asymptote will be

$$x+y = \lim_{\substack{x \rightarrow \infty \\ y \rightarrow 0}} a(x^4+y^4), \text{ in the direction of } x+y$$

$$\text{Let } x = 1/t \quad \therefore y = x = -1/t$$

$$\text{Then } x+y = \lim_{t \rightarrow 0} \frac{\frac{1}{t^4} + a^4}{t^4}$$

$$= \lim_{t \rightarrow 0} \frac{\frac{1}{t^4} + \frac{1}{t^4}}{1 + a^4 t^4}$$

$$= \frac{9}{2}$$

Thus the asymptote is

$$x+y = \frac{9}{2}$$

$$\Rightarrow 2x+2y-9=0$$

Find the asymptotes of the curve.

$$(x+y)^2(x+xy+y) = 2x^2y + 2y^2 + 2xy$$

Simplifying

The given equation is of the form

$$(x+2y+2)F_1(y) + F_0(y) = 0$$

thus one of the asymptotes is
 $x+2y+2=0$, and the other
two asymptotes are given by

$$x+y = \pm \sqrt{\lim_{t \rightarrow 0} \frac{y_t + 2}{x_t + 2}}$$

To evaluate this limit,

Put $x=-y$ and $y=t$

Then $x = -y = t = \infty$, when $t \rightarrow 0$

$$\therefore x+y = \pm \sqrt{\lim_{t \rightarrow 0} \frac{y_t + 2}{x_t + 2}} = \pm \sqrt{-y_t + 2}$$

$$= \pm \sqrt{\frac{4t^2 + 2t}{4t^2 + 1}} = \pm \sqrt{\frac{4t^2}{4t^2 + 1}} = \pm \sqrt{1} = \pm 1$$

Hence the asymptotes are

$$x+2y+2=0$$

$$x+2y+2\sqrt{2}=0$$

$$\text{and } x+2y-2\sqrt{2}=0$$

Find the asymptotes of the cubic Curve
 $x^3 + 4x^2y + 4xy^2 + 5y^3 + 15xy + 10y^2 - 2y + 7 = 0$

Here Equations of the Curve.

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$$x^3 + 4x^2y + 4xy^2 + 5y^3 + 15x^2y + 10y^2 - 2y + 1 = 0$$

$$\Rightarrow x(x^2 + 4xy + 4y^2) + 5(x^2 + 3xy + 2y^2) - 2y + 1 = 0$$

$$\Rightarrow x(x+2y)^2 + 5(x+y)(x+2y) - 2y + 1 = 0$$

The Asymptotes parallel to $x+2y = 0$ are obtained by

$$(x+2y)^2 + 5(x+2y) \lim(1+\frac{y}{x}) + \lim(-\frac{2y}{x} + \frac{1}{x}) = 0$$

When $x \rightarrow 0$ $\frac{y}{x} \rightarrow -\frac{1}{2}$

$$\Rightarrow (x+2y)^2 + 5(x+2y)(1 - \frac{1}{2}) + [-2(-\frac{1}{2}) + 0] = 0$$

$$\Rightarrow (x+2y)^2 + 5(\frac{1}{2}(x+2y)) + 1 = 0$$

$$\Rightarrow 2(x+2y)^2 + 5(x+2y) + 2 = 0$$

$$\Rightarrow 2(x+2y)^2 + 4(x+2y) + x+2y+2 = 0$$

$$\Rightarrow 2(x+2y)^2 + 4(x+2y) + (x+2y+2) = 0$$

$$\Rightarrow 2(x+2y)(x+2y+2) + (x+2y+2) = 0$$

$$\Rightarrow (x+2y+2)(2x+4y+1) = 0$$

$$\Rightarrow x+2y+2=0 \quad 2x+4y+1=0$$

In the given curve we find.

that x^3 is present - but there is no term containing y^3 .

Hence there will be no asymptote

parallel to the x -axis

To get the asymptote parallel to y -axis

coefficient of $y^2 = 0$

$$2 - 4x + 10 = 0 \Rightarrow 2x + 5 = 0$$

Hence the required asymptotes are

$$x+2y+2=0 \quad 2x+4y+1=0 \quad 2x+5=0$$